DOT PUZZLES

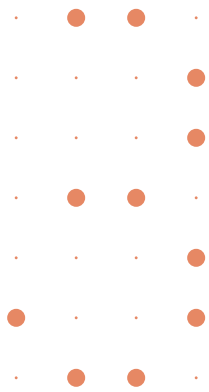
STRATEGIES

1. Start with squares.

Squares have the most "restrictions" because they must have four right angles and four equal sides. So, there are fewer squares than other types of quadrilaterals.

How can we start the puzzle below?

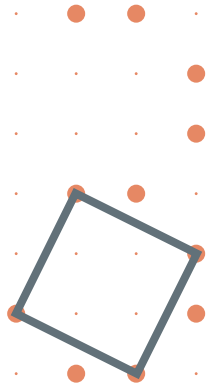
1 square
1 parallelogram



There are many different parallelograms we can draw using four of these points. But, there is only one square.

We connect four points to make a square as shown below.

1 square
1 parallelogram



Then, we look for a way to make a parallelogram that does not use any of these points.

2. Some shapes are more than one type of quadrilateral.

Every square is also both a rectangle and a rhombus, since every square has four right angles and four equal sides.

So, if a puzzle asks us to find rhombuses or rectangles, squares count, too!

When this happens, the puzzle will always include a note as a reminder.

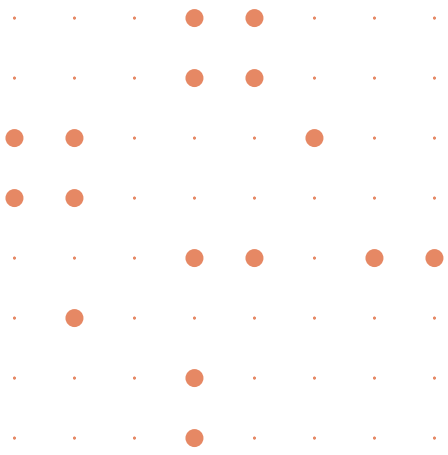
3. Count the total number of dots in the puzzle.

Since every quadrilateral has four corners, every quadrilateral uses exactly four dots.

So, the number of dots in a puzzle tells us how many dots are not used.

How many of the dots in the puzzle below will not be part of a square?

4 squares



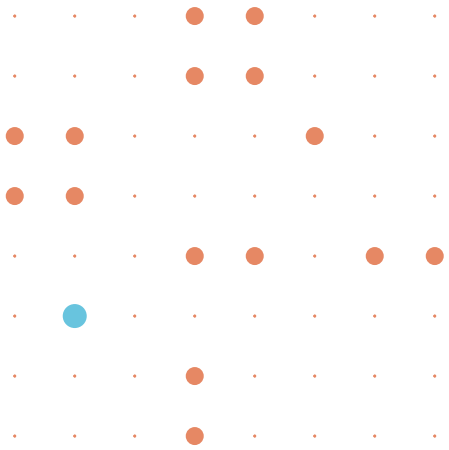
We count a total of 16 dots. To draw 4 squares, we must use $4 \times 4 = 16$ dots. So, every dot in the puzzle must be the corner of a square!

4. Look for dots that can only be part of 1 shape.

In strategy 3, we learned that every dot in the puzzle below must be part of a square.

What square(s) can the blue dot below be a part of?

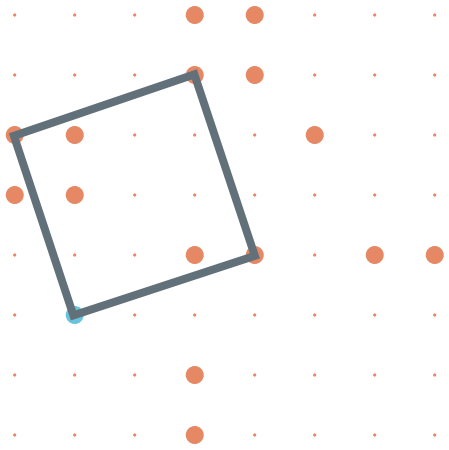
4 squares



There is only one square that can be drawn on the grid that uses the blue dot as a corner.

Since every dot in this puzzle must be used, we know this square must be part of the solution.

4 squares

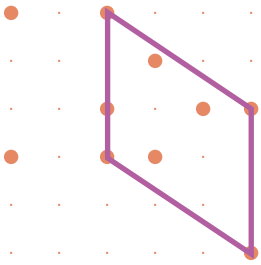


5. Stay organized and eliminate possibilities.

If there are limited possibilities for a point or shape, try them all. Try to rule out possibilities that don't work to avoid trying them again later. (Be careful! Don't rule anything out unless you're sure you haven't missed anything.)

Can this parallelogram be part of the solution to the puzzle below?

2 parallelograms (Neither can be a rectangle.)



There are 6 points left. We carefully check groups of 4 points to see if they make a parallelogram. One way to do this is by selecting two points to leave out, then checking if the remaining four points make a parallelogram.

The only parallelogram we can make using four of the remaining six points is a square, but this doesn't count because every square is also a rectangle (strategy 2).

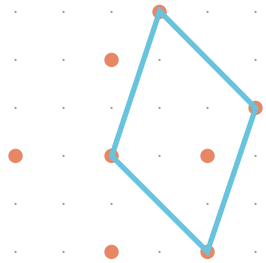
So, this parallelogram cannot be part of the solution to this puzzle. We erase it and make a note not to try it again.

6. Check for parallel sides.

We can find out if two sides are parallel by counting "hops" between dots on their sides.

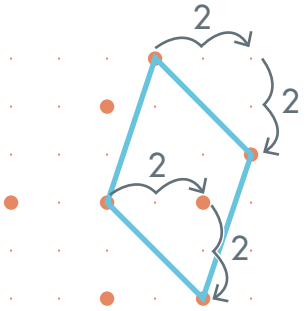
Sides that are parallel have the same pattern of hops in the same directions from one dot to the next.

Is the shape below a parallelogram?

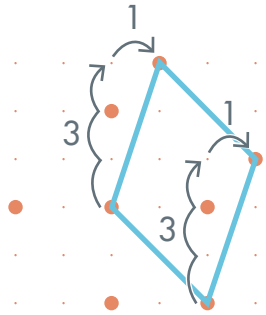


The shape *looks* like a parallelogram, but how can we tell? To find out, we count how many hops we need to get from each corner to the next.

For the sides below, we hop 2 dots right and 2 dots down to get from one corner to the other. So, they are parallel.



For the other two sides, we hop 3 dots up and 1 dot right to get from one corner to the other. So, they are also parallel.



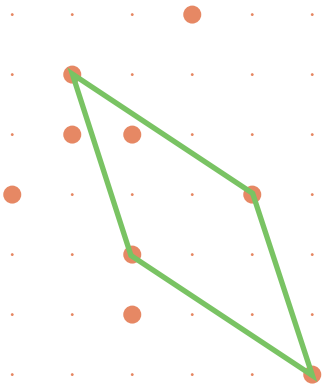
Since opposite sides are parallel, this quadrilateral is a parallelogram.

7. Check for equal side lengths.

In all of the dot puzzles in this book, we can find out if two lengths are equal by counting hops along their sides.

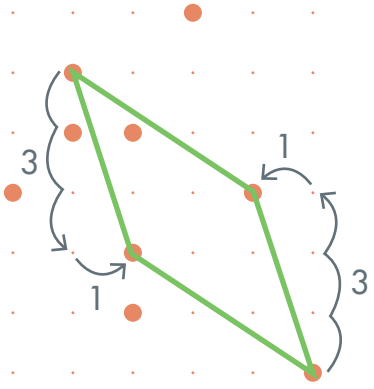
Sides that are the same length have the same pattern of hops from one end to the other, but not always in the same order or direction. For example, hopping 3 dots right and 2 dots up is the same distance as hopping 2 dots left and 3 dots down.

Is the green quadrilateral below a rhombus?

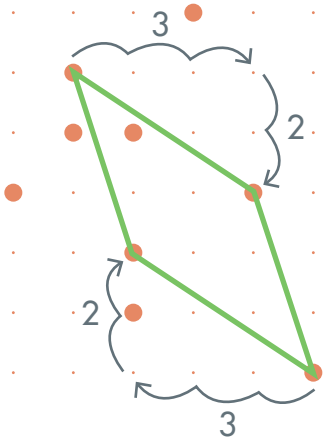


The shape *looks* like a rhombus, but how can we tell for sure? To compare the lengths of its sides, we count how many hops it takes to get from one corner to the next.

On two of the sides, we hop 3 dots one way and 1 dot another way to get from one corner to the other.



But for the other two sides, we hop 3 dots one way and 2 dots another way to get from one corner to the other.



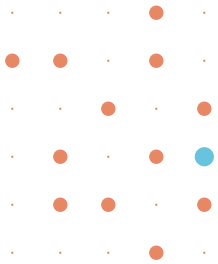
So, these sides are a little longer than the other two. Since the four sides are not all the same length, this shape is not a rhombus.

(We will learn how to find these lengths when we learn about the Pythagorean Theorem in Beast Academy 5D.)

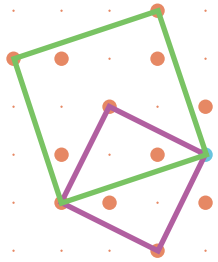
8. Check for squares.

We can count hops along sides to check squares, too.

Find all of the squares that have a corner on the blue dot below.

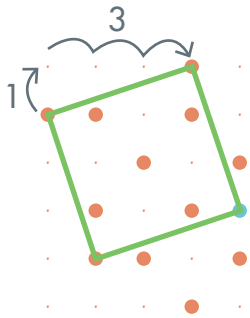


There are two squares we can make that have a corner on the blue dot.

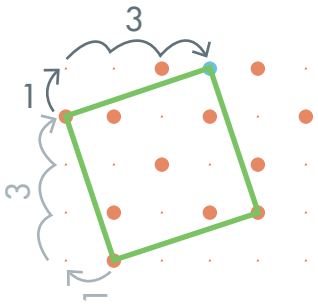


The pattern of hops along the sides of a square will always be the same number of hops, but rotated as we move around the square.

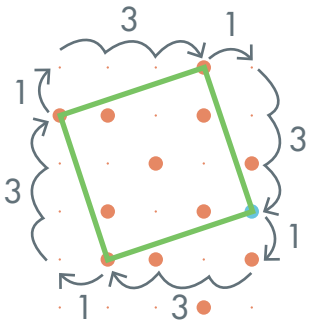
For example, from the top-left corner to the top-right corner of the green square below, we hop up 1, and right 3.



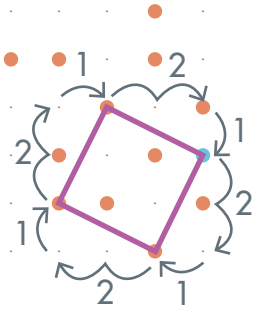
If we turn the grid on its side, we make the same hops to get to the next corner.



We can continue around the square.



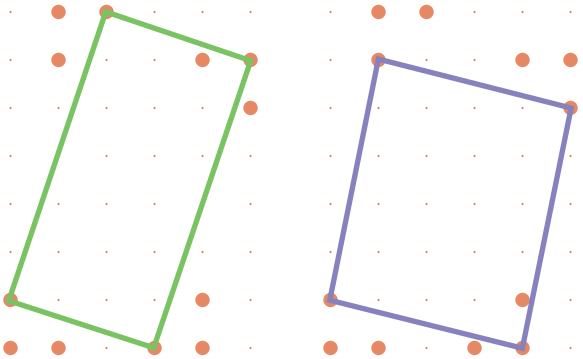
Similarly, we can find the pattern of hops for the smaller square below.



9. Check for right angles.

We can even count hops to see if two sides meet at a right angle.

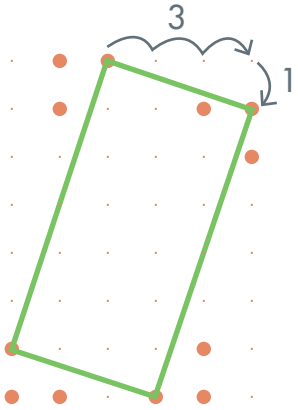
Is either quadrilateral below a rectangle?



We could use the corner of a piece of paper to see if the angles are right, but these angles are all really close!

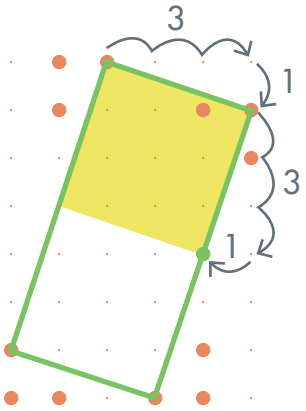
So, we look at the dots in the grid and use what we know about squares from strategy 8.

For the top side of the green quadrilateral, we go right 3 hops and down 1 hop to get from one corner to the other.

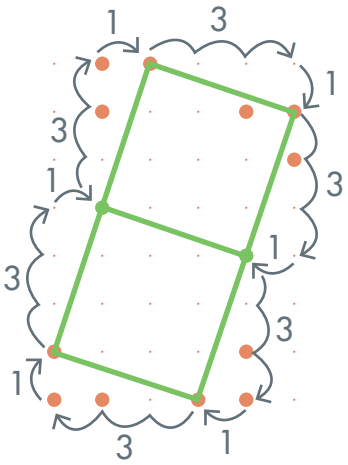


If we turn this pattern around the corner, going down 3 and left 1, we reach a point on the side of the rectangle.

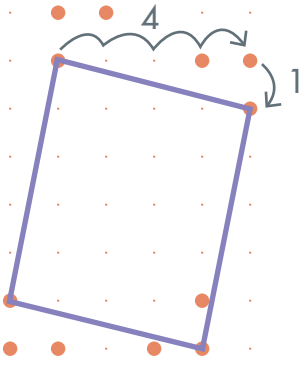
So, these dots make the corner of a square, and the top-right corner must be a right angle!



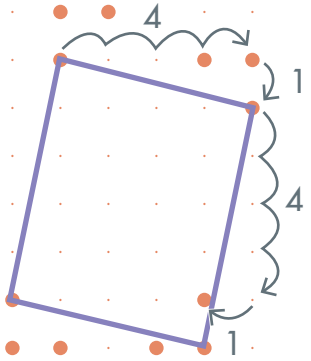
We can continue around the shape to see that the green quadrilateral can be made from two squares. So, it is a rectangle.



For the top side of the purple quadrilateral, we go right 4 hops and down 1 hop to get from one corner to the other.



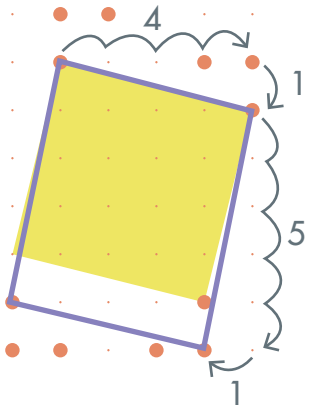
To make a right angle, we turn this pattern around the corner by going down 4 and left 1.



But, this point is not on a side of the purple quadrilateral.

From one corner to the other of a long side, we go down 5 and left 1. So, the top-right angle is a little bigger than a right angle.

We've drawn a square in the diagram below to show what a right angle would look like.

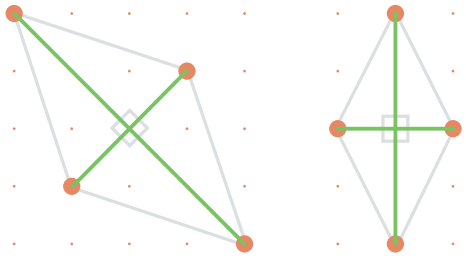


Since the purple quadrilateral does not have four right angles, it is not a rectangle.

10. Check for rhombuses.

Rhombuses have special properties that can help us find them. In a rhombus, the diagonals cross at a right angle. And, the diagonals split each other in half.

So, when looking for rhombuses, it often helps to look for diagonals like the pairs below.



11. Look for one pair of sides.

In any parallelogram, if one pair of opposite sides are parallel and the same length, then the other pair of sides will also be parallel and the same length.

So, when looking for parallelograms, we can look for just one pair of opposite sides. Look for pairs of dots that are separated by the same numbers of hops like the ones shown below.